CHAPTER # 4 THREE-PHASE INDUCTION MACHINES

1- Introduction (General Principles)

Generally, conversion of electrical power into mechanical power takes place in the ***rotating*** part of an electric motor. In DC motors, the electric power is ***conducted*** directly to the armature (*i.e.* rotating part) through brushes and commutator. Hence, in this sense, a DC motor is called a ***conduction*** motor. However, in AC motors, the rotor receives its electric power by ***induction*** in exactly the same way as the secondary of a 2-winding transformer receives its power from the primary. That is why such motors are known as ***induction*** motors. In fact, an induction motor can be treated as a ***rotating transformer*** *i.e.* one in which primary winding is stationary but the secondary is free to rotate.

2- Construction

An induction motor consists essentially of two main parts: **(*a*)** stator and **(*b*)** rotor.

(*a*) Stator

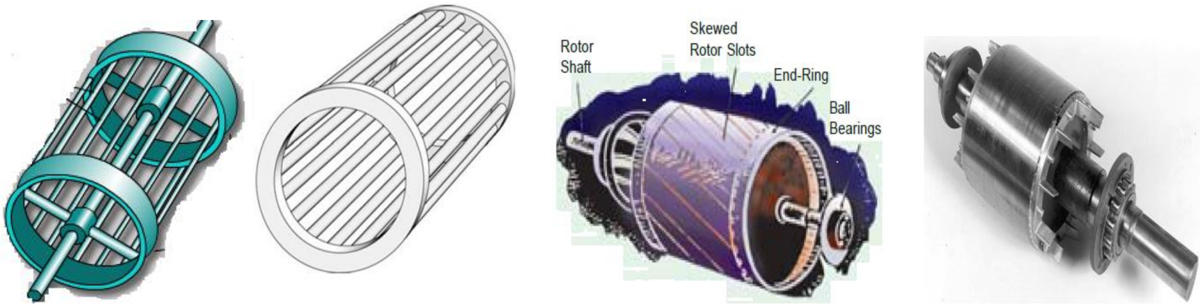
The stator of an induction motor is the same as that of a synchronous motor or generator. It is made up of a number of stampings, which are slotted to receive the windings as shown in Fig. 1. The stator carries a 3-phase winding and is fed from a 3-phase supply. It is wound for a definite number of poles, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and ***vice versa***. As explained in chapter 1, that the stator windings, when supplied with 3-phase currents, produce a magnetic flux, which is of constant magnitude and revolves (rotates) at synchronous speed ($Ns = 60\ f/P$). This revolving magnetic flux induces an e.m.f. in the rotor by mutual induction.

Fig. 1, Stator of three-phase Induction Machines

(*b*) Rotor

There are two main types of rotor:

(i) ***Squirrel-cage rotor*** **:** Motors employing this type of rotor are known as squirrel-cage induction motors. Almost 90 per cent of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction and is almost indestructible. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which are not wires but consist of heavy bars of copper, aluminium or alloys. One bar is placed in each slot, rather the bars are inserted from the end when semi-closed slots are used. The rotor bars are brazed or electrically welded or bolted to two heavy and solid short-circuiting end-rings, thus giving us, what is so called, a squirrel-cage construction (Fig. 2).



(a) (b) (c)

Fig. 2, Squirrel-cage Rotor

It should be noted that the ***rotor bars are permanently short-circuited on themselves,*** hence it is not possible to add any external resistance in series with the rotor circuit for starting purposes. The rotor slots are usually not quite parallel to the shaft but are purposely given a slight skew (Fig. 2-a). This is useful in two ways :

- it helps to make the motor run quietly by reducing the magnetic hum and
- it helps in reducing the locking tendency of the rotor *i.e*. the tendency of the rotor teeth to remain under the stator teeth due to direct magnetic attraction between the stator and rotor.

In small motors, another method of construction of the squirrel-cage rotor is used. It consists of placing the entire rotor core in a mould and casting all the bars and end-rings in one piece. The metal commonly used is an aluminum alloy (Fig. 2-b).

Another form of rotor consists of a solid cylinder of steel without any conductors or slots at all. The motor operation depends upon the production of eddy currents in the steel rotor (Fig. 2-c).

***(ii) Wound rotor*:** Motors employing this type of rotor are variously known as ‘wound’ motors or as ‘slip-ring’ motors as shown in Fig. 3.

Fig. 3, Wound or Slip-Ring Rotor

This type of rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phase. The three phases are star connected internally. The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them as shown in Fig 3. These three brushes are further externally connected to a 3-phase star-connected rheostat as shown in Fig. 4. This insert an additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor.

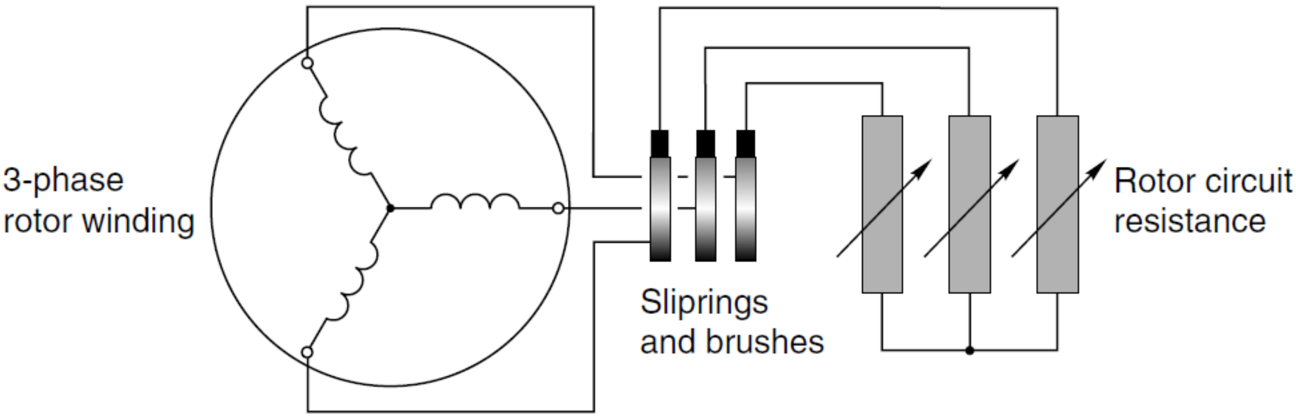


Fig. 4, External Y-connected resistance

When running under normal conditions, the ***slip-rings are automatically short-circuited*** by means of a metal collar, which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and the wear. Hence, it is seen that under normal running conditions, the wound rotor is short-circuited on itself just like the squirrel-cage rotor.

3. Theory of Operation

3.1. Motor Mode

Induction machine does not require electrical connection to the rotor windings. Instead, the rotor windings are short circuited. The 3-phase stator (armature) windings, are fed by a 3-phase supply then a magnetic flux of constant magnitude, but rotating at synchronous speed (N_S), is established. This flux passes through the air-gap, and cuts the rotor conductors which, as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor conductors, an e.m.f. is induced in the short-circuited conductors, according to Faraday's laws of electro-magnetic induction. ***The frequency of the induced e.m.f. is the same as the supply frequency.*** Its magnitude is proportional to the relative velocity between the flux and the conductors and its direction is given by Fleming's Right-hand rule. Since the rotor bars or conductors form a closed circuit, rotor current is produced whose direction, as given by Lenz's law, is such as to oppose the cause producing it. In this case, the cause which produces the rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductors. Hence, to reduce the relative speed, the rotor starts running in the ***same*** direction as that of the flux and tries to catch up with the rotating flux.

Figure 5-*a*, shows the stator field which is assumed to be rotating clockwise. The relative motion of the rotor with respect to the stator is ***anticlockwise.*** By applying Right-hand rule, the direction of the induced e.m.f. in the rotor is found to be outwards. Hence, the direction of the flux due to rotor current *alone*, is as shown in Fig. 5-*b*. Now, by applying the Left-hand rule, or by the effect of combined field shown in Fig. 5-*c*, it is clear that the rotor conductors experience a force tending to rotate them in clockwise direction. Hence, the rotor is set into rotation in the same direction as that of the stator (or field) flux.

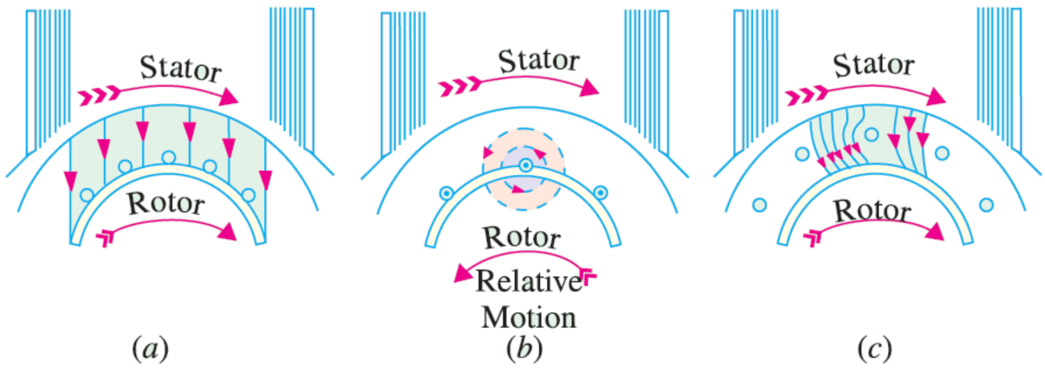


Fig. 5, Direction of rotor motion

3.2 Slip

In practice, in motor mode, the rotor never succeeds in 'catching up' with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f., no rotor current and so no torque to maintain rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field. The difference in speeds depends upon the load on the motor. The difference between the synchronous speed *Ns* and the actual speed *N* of the rotor is known as the ***slip*** **(S)**, that can be expressed as a percentage of the synchronous speed as follows:

$$\%\,\text{slip} \quad s = \frac{N_s - N}{N_s} \times 100$$

Sometimes, *Ns* − *N* is called the ***slip speed.***

Obviously, rotor (or motor) speed is $N = Ns\,(1 - s)$.

It may be kept in mind that revolving flux is rotating synchronously, relative to the stator (*i.e*. stationary space) but at slip speed relative to the rotor.

3.3. Generator Mode

Induction machines are usually operated in motor mode. So they are usually called induction motors. In normal motor operation, stator flux rotation is faster than the rotor rotation. This causes the stator flux to induce rotor currents, which create a rotor flux with magnetic polarity opposite to stator. In this way, the rotor is dragged along behind stator flux, at a value equal to the slip.

On the other hand, in case of generator mode, a prime mover (turbine, engine) drives the rotor above the synchronous speed. The stator flux still induces currents in the rotor, but since the opposing rotor flux is now cutting the stator coils, an active current is produced in stator coils, and the motor now operates as a generator, sending power back to the electrical grid.

Therefore, the induction generators are not self-exciting. This means, they need an initial electrical supply to produce the rotating magnetic flux. Practically, an induction generator will often self start due to residual magnetism. The rotating magnetic flux from the stator induces currents in the rotor, which also produces a magnetic field. If the rotor turns slower than the rate of the rotating flux, the machine acts like an induction motor. If the rotor is turned faster, it acts like a generator, producing power at the synchronous frequency.

3.4 Frequency of Rotor Current

When the rotor is stationary, the frequency of rotor current is *the same as the supply frequency*. But when the rotor starts revolving, then the frequency depends upon the relative speed or on slip speed. Let the stator or supply frequency is f_1 also let the frequency of the rotor current at any slip-speed is f_2 . Then

$$slip\ speed = N_s - N = \frac{60\, f_2}{P}$$

$$N_s = \frac{60\, f_1}{P}$$

Dividing the above two equations,

$$\frac{N_s - N}{N_s} = \frac{f_2}{f_1} = S$$

$$rotor\ frequency\ f_2 = S\, f_1$$

When the rotor current flows through the individual phases of the rotor winding, produces the rotor magnetic fields. These individual rotor magnetic fields produce a combined rotating magnetic field, whose speed relative to rotor is

$$\frac{60\, f_2}{P} = \frac{60\, S f_1}{P} = S\, N_s$$

Example (1): *A slip-ring induction motor runs at 290 r.p.m. at full load, when connected to 50-Hz supply. Determine the number of poles and slip.*

Since N is 290 rpm; Ns has to be somewhere near it, N_S = 300 rpm.

Therefore 300 = 60 × f_1/P =60×50/P. Hence, P = 10 and total number of poles is 2P = 20. ∴S = (300 − 290)/300 = 3.33%

Example (2): *The stator of a 3-φ induction motor has 3 slots per pole per phase. If supply frequency is 50 Hz, if the number of poles is double the coil groups; calculate:* **(*i*)** *number of stator slots* **(*ii*)** *speed of the rotating stator flux (or magnetic field).*

$2P = 2Q = 2 \times 3 =$ **6 poles**

Total No. of slots = 3 slots/pole/phase × 6 poles × 3 phases = **54**

(*ii*) $Ns = 60\ f/P = 60 \times 50/3 =$ **1000 r.p.m.**

Example (3): *A 4-pole, 3-phase induction motor operates from a supply whose frequency is 50 Hz. Calculate :* **(*i*)** *the speed at which the magnetic field of the stator is rotating.* **(*ii*)** *the speed of the rotor when the slip is 0.04.* **(*iii*)** *the frequency of the rotor currents when the slip is 0.03.* **(*iv*)** *The frequency of the rotor currents at standstill.*

(*i*) Stator field revolves at synchronous speed, given by

$Ns = 60\ f_1/P = 60 \times 50/2 =$ **1500 r.p.m.**

(*ii*) rotor (or motor) speed, $N = Ns\ (1 - s) = 1500(1 - 0.04) =$ **1440 r.p.m.**

(*iii*) frequency of rotor current, $f_2 = sf_1 = 0.03 \times 50 = 1.5$ c.p.s = **1.5 Hz**

(*iv*) Since at standstill, $s = 1$, $f_2 = sf_1 = 1 \times f_1 =$ **50Hz**

Example (4): *A 3-φ induction motor is wound for 4 poles and is supplied from 50-Hz system. Calculate* ***(i)*** *the synchronous speed* ***(ii)*** *the rotor speed, when slip is 4% and* ***(iii)*** *rotor frequency when rotor runs at 600 rpm.*

(*i*) $Ns = 60\ f_1/P = 60 \times 50/2 = 1500$ rpm

(*ii*) rotor speed, $N = Ns\ (1 - s) = 1500\ (1 - 0.04) =$ **1440 rpm**

(*iii*) when rotor speed is 600 rpm, slip is $s = (Ns - N)/Ns = (1500 - 600)/1500 = 0.6$

rotor current frequency, $f_2 = sf_1 = 0.6 \times 50 =$ **30 Hz**

Example (5): *A 12-pole, 3-phase alternator driven at a speed of 500 r.p.m. supplies power to an 8-pole, 3-phase induction motor. If the slip of the motor, at full-load is 3%, calculate the full-load speed of the motor.*

Let N = actual motor speed; Supply frequency, $f_1 = 6 \times 500/60 = 50$ Hz.

Synchronous speed $Ns = 60 \times 50/4 = 750$ r.p.m.

Slip $s = 0.03 = (N_s - N)/N_s = (750 - N)/750$

Rotor speed $N = 750 - 0.03 \times 750 =$ **727.5 r.p.m.**

4. Equivalent Circuit per phase for Induction Motor

we can derive the equivalent circuit for one phase of the 3-phase induction machine, with the understanding that the voltages and currents in the remaining phases can be found simply by an appropriate phase shift of those of the phase under study (±120°).

First, consider conditions of operation of induction motor whose stator is supplied by a terminal voltage V_1. The stator terminal voltage differs from the generated back emf (E_1) by the voltage drop in the stator leakage impedance $Z_1 = R_1 + j X_1$.

As shown in Fig. 6, the stator current can be resolved into two components: a load component (I_2) and an exciting (magnetizing) component (I_φ). The load component produces the rotor mmf. The exciting component is the additional stator current required to create the resultant air-gap flux.

The magnetizing current (I_φ) can be resolved into a core-loss component I_c in phase with E_1 and a magnetizing component I_m lagging E_1 by 90°. In the equivalent circuit, the magnetizing current can be accounted for by means of a shunt branch, formed by a *core-loss resistance* R_c and a *magnetizing reactance* X_m in parallel.

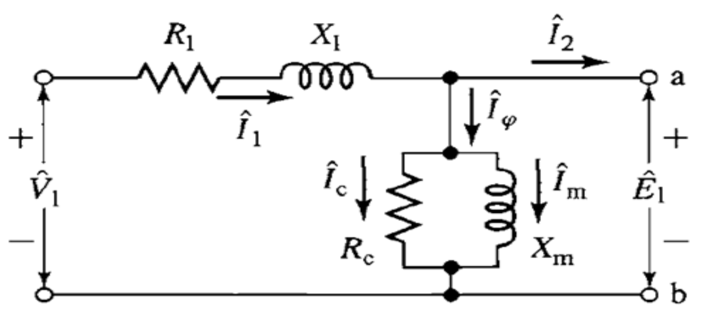


Fig. 6, Stator equivalent circuit of one phase of induction motor

Where

V_1 Stator line-to-neutral terminal voltage

E_1 Back emf (line-to-neutral) generated by the resultant air-gap flux

I_l Stator current

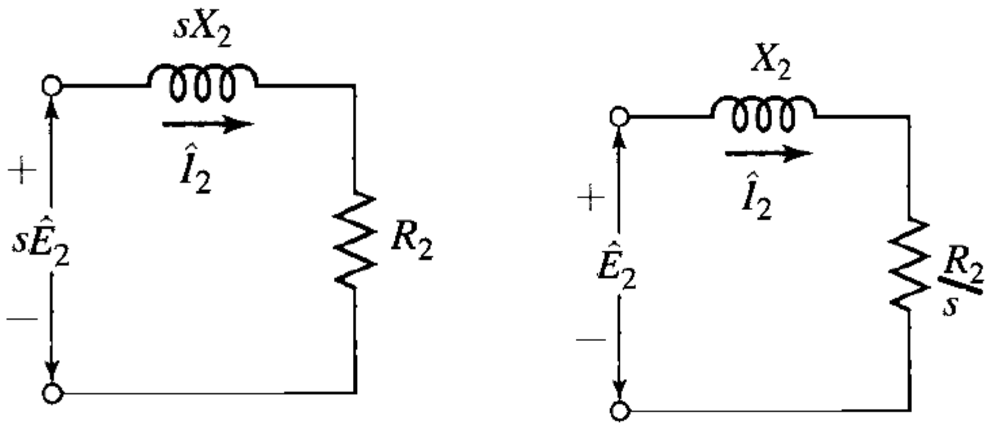
R_1 Stator effective resistance

X_1 Stator leakage reactance

R_c Core loss resistance

X_m magnetizing branch reactance

Similarly, the equivalent circuit of the rotor of induction motor is shown in Fig. 7. The rotor of an induction machine is short-circuited, and hence the impedance seen by induced voltage is simply the rotor short-circuit impedance. Both of the rotor leakage reactance SX_2 and back emf SE_2 depend on the rotor frequency. But the rotor effective resistance R_2 doesn't depend on the frequency (Fig. 7-a). Dividing all elements of the equivalent circuit by S, we can obtain the circuit shown in Fig (7-b).



(a) Rotor equivalent circuit (b) Dividing all elements by s

Fig. 7, Rotor equivalent circuit of one phase of induction motor

Where R_2 rotor effective resistance

X_2 rotor leakage reactance

I_2 rotor current

E_2 Back emf (line-to-neutral) generated by the resultant air-gap flux

To study the performance of induction motor, it is recommended to refer the rotor circuit to the stator circuit similar to that of the transformer as shown in Fig. 8.

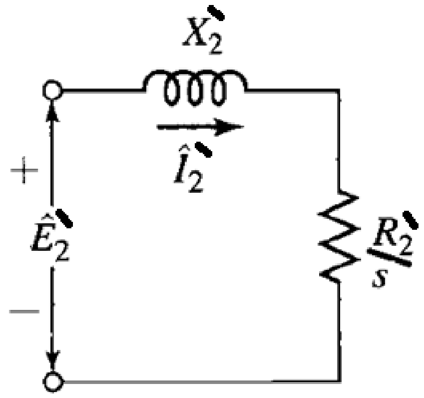


Fig. 8, Rotor equivalent circuit referred to stator circuit

Therefore, the overall exact equivalent circuit of the induction motor viewed from the stator is shown in Fig. 9.

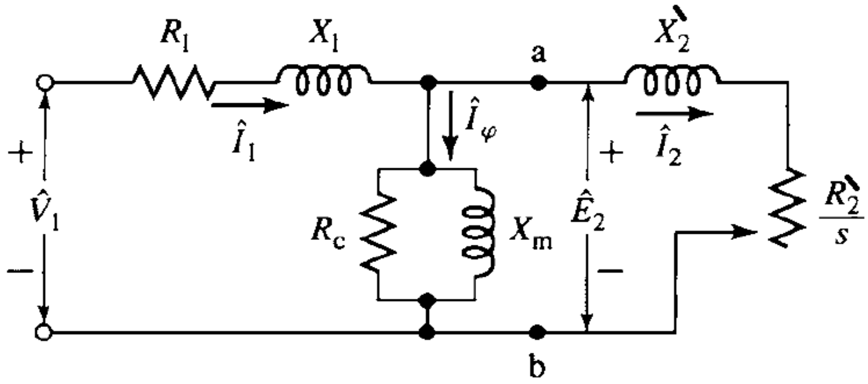


Fig. 9, Exact equivalent circuit of one phase of a 3-phase induction motor

The resistance (R_2'/S) can be divided into two resistances R_2' and $R_2'(1\text{-}S)/S$, where R_2' represents the referred rotor resistance and $R_2'(1\text{-}S)/S$ represents the mechanical load connected to the motor shaft as shown in Fig. 10.

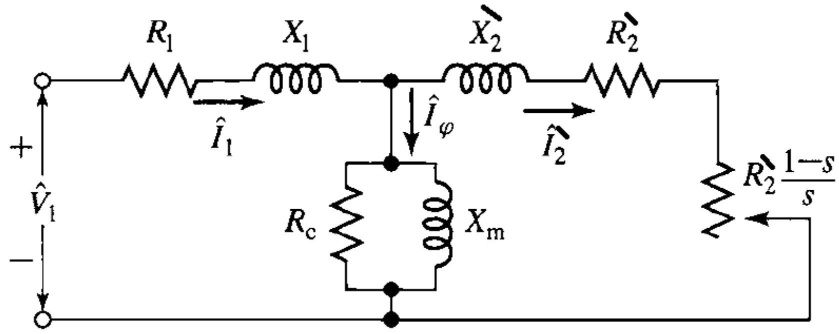


Fig. 10, Exact equivalent circuit that represents mechanical load of induction motor

If the magnetizing branch that contain R_c and X_m is moved towards the terminal voltage, we can obtain the approximate equivalent circuit as shown in Fig. 11.

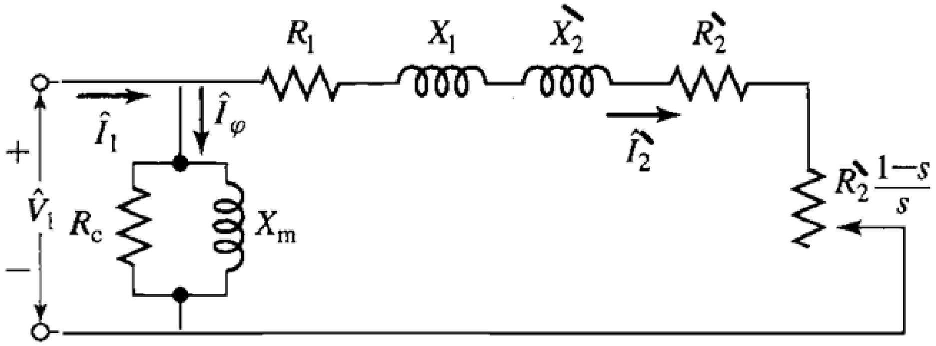


Fig. 11 Approximate equivalent circuit of induction motor

Using circuits lows, we can calculate the stator current, rotor current and the magnetizing branch current. The three phase powers can be calculated as follows:

Input power P_{in} $= 3\ V_1\ I_1 \cos(\phi)$

Where ϕ is the angle between the terminal voltage and stator current

Stator Copper loss (P_{cu1}) $= 3\ I_1^2\ R_1$

Stator Iron loss (P_c) $= 3\ I_c^2\ R_c$

Rotor Copper loss (P_{cu2}) $= 3\ I_2'^2\ R_2'$

Mechanical output power (gross) P_m $= 3\ I_2'^2\ R_2'(1\text{-}S)/S$

Mechanical output power (net) $= 3\ I_2'^2\ R_2'(1\text{-}S)/S - P_{f+w}$

Where P_{f+w} is mechanical losses represented by friction and windage.

The air gap power (P_g) is considered as the sum of Rotor Copper loss (P_{cu2}) and Mechanical output power (gross),

The air gap power (P_g) $= 3\ I_2'^2\ R_2' + 3\ I_2'^2\ R_2'(1\text{-}S)/S = 3\ I_2'^2\ R_2'/S$

The power flow from input electrical power to the output mechanical power is shown in Fig. 12.

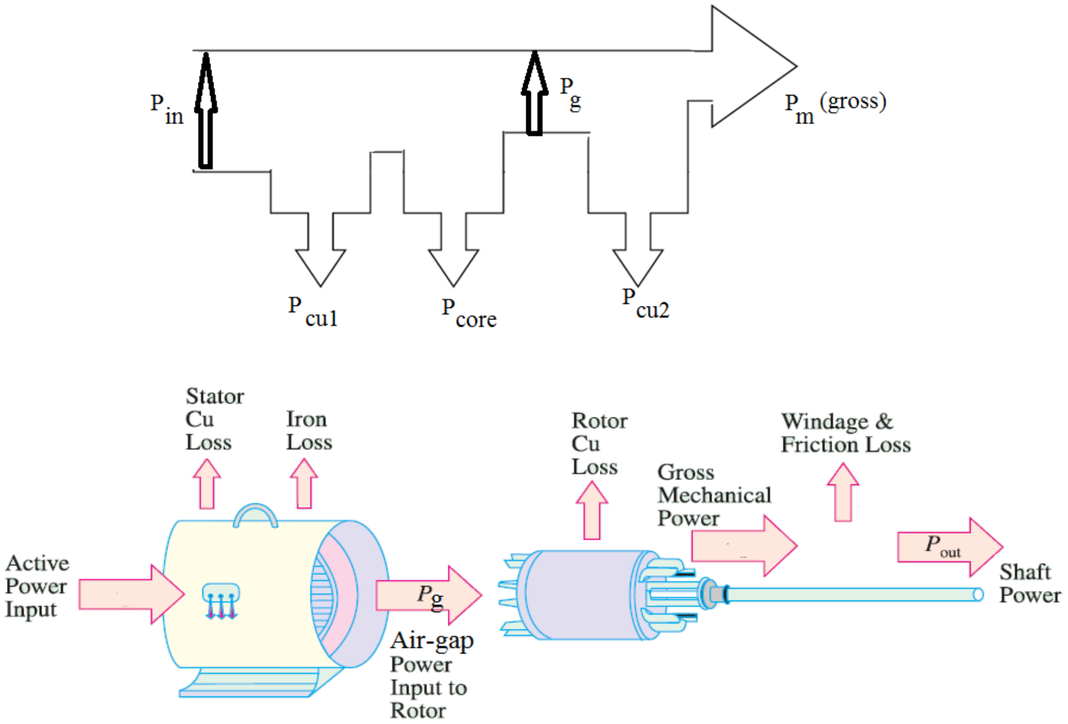


Fig. 12, Power flow in induction motor

The ratio between P_{cu2} and P_m and P_g can be given as follows:

Rotor Copper loss (P_{cu2})	**Mechanical output power (P_m)**	**The air gap power (P_g)**
1	$\frac{1-S}{S}$	$\frac{1}{S}$
S	1 – S	1

The stator efficiency = P_g/P_{in}

The rotor efficiency = P_{out}/P_g

The motor efficiency = rotor efficiency × stator efficiency = P_{out} / P_{in} = P_{out} / (P_{out} + P_{losses}) Where, P_{losses} is the sum of all losses occur in the stator and rotor.

Example (6)

The power input to the rotor of 440 V, 50 Hz, 6-pole, 3-phase, induction motor is 80 kW. The rotor electromotive force is observed to make 100 complete alterations per minute. Calculate

***(i)** the slip, **(ii)** the rotor speed, **(iii)** rotor copper losses per phase.*

Solution:

Rotor alternation per minutes = 100

Rotor alternation per second (f_2) = 100/60 = 1.6667 = Sf_1

S = 1.6667/50 = 0.03333 = 3.33%

N_s = 60 × 50 / 3 = 1000 rpm

N = N_s(1-s) = 1000 (1-0.0333) = 966.67 rpm

The rotor input power is the air gap power = 80 kW

The rotor copper loss P_{cu2} = S P_g = 0.0333 × 80000 = 2.667 kW (for the 3 phase)

= 2667/3 = 888.8 W

<u>Example (7)</u>

A three-phase, Y-connected, 220-V (line-to-line), 7.5-kW, 60-Hz, 6-pole induction motor has the following parameter values in Ω/phase referred to the stator:

R_1 = 0.294Ω　　R_2' = 0.144 Ω　　R_c = 415 Ω

X_1 = 0.503 Ω　　X_2' = 0.209 Ω　　X_m = 13.25 Ω

The total friction and windage losses may be assumed to be constant at 403 W, independent of load. For a slip of 2 %, and based on approximate equivalent circuit, compute the rotor speed, output torque and power, stator current, power factor, iron loss, electromagnetic torque and efficiency when the motor is operated at rated voltage and frequency.

Solution

Refer to the approximate equivalent circuit shown in Fig. 11,

S = 0.02　　$N_s = 60\times f / P = 60\times 60 / 3 = 1200$ rpm

** The rotor speed (N) = N_s (1-S) = 1200 (1-0.02) = 1176 rpm #

$$\frac{R_2'}{S} = \frac{0.144}{0.02} = 7.2\ \Omega$$

Z = (R_1 + R_2'/S) + J(X_1+X_2') = 7.494+J0.712 = 7.5277∟5.4273 Ω

$$I_2' = \frac{V_1}{Z} = \frac{220/\sqrt{3}}{7.5277\angle 5.4273} = 16.8732\angle -5.4273\ A$$

Z_m = R_c // JX_m = (R_c × JX_m) / (R_c + JX_m) = 13.2433∟88.1713 Ω

$$I_{\phi} = \frac{V_1}{Z_m} = \frac{220/\sqrt{3}}{13.2433\angle 88.1713} = 9.5911\angle -88.1713\ A$$

** The stator current can be obtained as: $I_1 = I_2' + I_\phi = 20.4346\angle -33.1761$ A

** The input P.F. is considered as cosine the angle between the input voltage and stator current i.e. cos(33.1761) = 0.837 lag.

The output mech. Power (Gross) = $3\ I_2'^2\ R_2'(1-S)/S = 3\times(16.8732)^2\times 0.144(1-0.02)/0.02$

= 6026.633 W

** The output mech. Power (Net) = The output mech. Power (Gross) – P_{f+w}

= 6026.633 – 403 = 5623.633 W

$\omega = 2\pi\ N/60 = 2\pi \times 1176 / 60 = 123.1504$ rad/s

** the developed (output) torque (T) = The output mech. Power (Net)/ω

= 5623.633 / 123.1504 = 45.6647 N.m

** Air-gap power (P_g) = $3\ I_2'^2\ R_2'/S = 3\times(16.8732)^2\times 0.144/0.02 = 6149.6254$ W

$\omega_S = 2\pi\ N_S/60 = 2\pi \times 1200 / 60 = 125.6637$ rad/s

** electromagnetic torque (T_{em}) = Air-gap power (P_g) / ω_S

= 6149.6254 / 125.6637 = 48.9372 N.m

Iron loss (P_{iron}) = $3\times V_1^2 / R_c = 3\times(220/\sqrt{3})^2/415 = 116.6265$ W

Input power (P_{in}) = $3\ V_1\ I_1 \cos(\phi) = 3\times (220/\sqrt{3}) \times 20.4346 \times 0.837 = 6517.408$ W

Efficiency = P_{out} / P_{in} = 5623.633 / 6517.408 = 86.286 %

Example (8):

A 100-kW (net output), 3300-V, 50-Hz, 3-phase, star-connected induction motor has a synchronous speed of 500 r.p.m. The full-load slip is 1.8% and full-load power factor is 0.85 lag. If the stator copper loss is 2440 W, the iron loss is 3500 W, and the friction and windage loss is 1200 W.

Calculate **(i)** Rotor copper loss and the gross mechanical power
(ii) Rotor efficiency, stator efficiency, and motor efficiency
(iii) Input line current

P_{mech}(Gross) = 100000+1200 = 101200 W

$$P_g = \frac{P_{mech}(Gross)}{1-S} = \frac{101200}{1-0.018} = 103055\ W$$

$P_{cu2} = P_g - P_{mech}$ (Gross) = 103055 – 101200 = 1855 W

$P_{in} = P_g + P_{iron} + P_{cu1}$ = 103055+3500+2440=108995 W

$$rotor\ efficiency = \frac{P_{out}(net)}{P_g} = \frac{100000}{103055} \times 100\% = 97.0356\%$$

$$stator\ efficiency = \frac{P_g}{P_{in}} = \frac{103055}{108995} \times 100\% = 94.55\%$$

$$motor\ efficiency = \frac{P_{out}(net)}{P_{in}} = \frac{100000}{108995} \times 100\% = 91.747\%$$

$$I_a = \frac{108995}{\sqrt{3} \times 3300 \times 0.85} = 22.434\ A$$

Example (9)

*A 400 V, 3-phase, 50 Hz, 4-pole, star-connected induction-motor takes a line current of 10 A with 0.86 p. f. lagging. If the total stator losses are 5 % of the its input. Rotor copper losses are 4 % of the rotor input, and mechanical losses are 3 % of the input of the rotor. Calculate **(i)** slip and rotor speed, **(ii)** torque developed in the rotor, and **(iii)** shaft-torque. **(iv)** motor efficiency*

Input to motor (P_{in}) = √3 × 400 × 10 × 0.86 = 5958 Watts

Total stator losses ($P_{cu1} + P_{iron}$) = 0.05 × 5958 = 298 Watts

Stator Output = 5958 − 298 = 5660 Watts

Rotor input (P_g) = Stator Output = 5660 Watts

Rotor Copper-loss (P_{cu2}) = 0.04 × 5660 = 226.4 Watts

Mechanical losses (P_{f+w}) = 0.03 × 5660 = 169.8 Watts

Mechanical output (Net) = 5660 − (226.4 + 169.8) = 5264 Watts

(i) slip, *s* = Rotor-copper-loss / rotor-input = 4 %

Synchronous speed, *Ns* = 60 × *f*/*P* =1500 rpm

Rotor speed, $N = Ns\,(1 - s) = 1500\,(1 - 0.04) = 1440$ rpm

(ii) Let the torque developed in the rotor = T_d

ω_r Angular speed of rotor = 2π N/60 = 150.72 rad/sec

ω_s Angular speed of flux = 2π N_s/60 = 157 rad/sec

Mechanical output (Gross) = Rotor input − Rotor-copper-loss

= 5660 − 226.4 = 5433.6 watts

the developed torque can be obtained either by:

T_d = 5433.6 / 150.72 = 36.05 Nw-m

T_d = 5660 / 157 = 36.05 Nw-m

(iii) Shaft-torque = T_m = P_m (Net) /ω_r

= 5264 / 150.72 = 34.93 Nw-m.

(iv) motor Efficiency = P_m(net)/P_{in} = 5264/5958 = 88.35%

5. Torque-Speed Characteristics

If there is no rotational loss and the rotor copper loss is also ignored, the developed torque and the electromagnetic torque are equal. There are two ways to calculate that torque

$$T(developed) = \frac{Mech.power\ (gross)}{\omega} = \frac{3\,{I_2'}^2\,R_2'(1-S)/S}{\omega}$$

$$T(electromagnetic) = \frac{Air - gap\ power}{\omega_S} = \frac{3\,{I_2'}^2\,R_2'/S}{\omega/(1-s)} = \frac{3\,{I_2'}^2\,R_2'(1-S)/S}{\omega}$$

Considering the approximate equivalent circuit,

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + \frac{R_2'}{S}\right)^2 + (X_1 + X_2')^2}}$$

$$T = \frac{3\,{V_1}^2\,\frac{R_2'}{S}}{\omega_S\left\{\left(R_1 + \frac{R_2'}{S}\right)^2 + (X_1 + X_2')^2\right\}}$$

From the above equation, we can plot the torque-slip (T-S) relation at different values of R_2' as shown in Fig. 13.

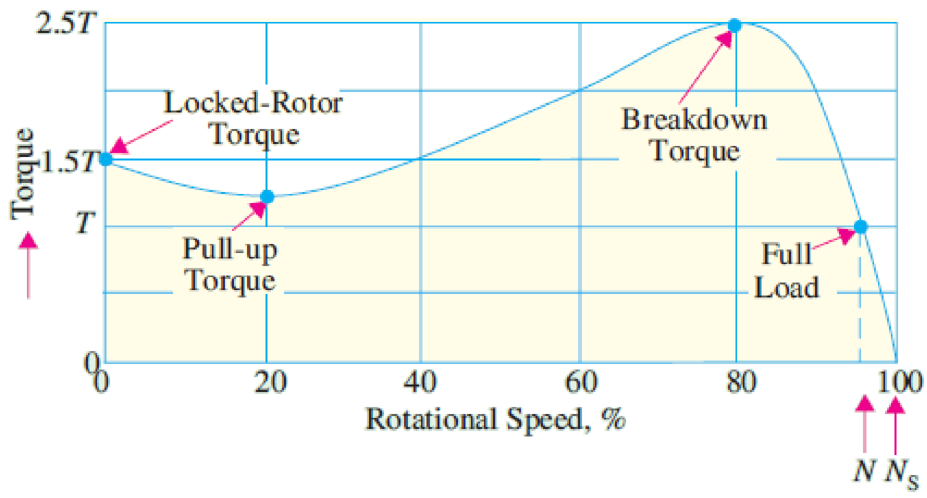


Fig. 13, Torque – speed characteristics of induction motor

It is clear that, for low values of slip, the torque/slip curve is approximately a straight line. the nominal (full load - rated) torque occurred at speed N and the torque is 1 pu. As slip increases (for increasing load on the motor), the torque also increases and becomes maximum. This torque is known as '***pull-out***' or '***breakdown***' or '***maximum***' torque T_m, and its value is 2.5pu. As the slip further increases (*i.e*. motor speed falls) and the torque is decreased until the rotor is at standstill (S=1) and the torque is called the starting torque T_{st}, and its value is 1.5 pu.

It is obvious that, the torque/slip curve is a rectangular hyperbola after the maximum torque. So, we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. The circuit-breakers will be tripped open if the circuit has been so protected. In fact, the stable operation of the motor lies between the values of $s = 0$ and that corresponding to maximum torque that called S_m.

The torque developed by a conventional 3-phase motor depends on its speed but the relation between the two cannot be represented by a simple equation. It is easier to show the relationship in the form of a curve (Fig. 13).

6. Maximum Torque

As we said before, $T=P_g/\omega_S$. Since the synchronous speed is constant, the maximum torque (T_m) occurs at the same slip as maximum air-gap power. This slip is called (S_m) and can be obtained as:

$$S_m = \frac{R_2'}{\sqrt{(R_1)^2 + (X_1 + X_2')^2}}$$

To obtain the expression for the maximum torque (T_m), substituting the value of S_m in the torque equation,

$$T_m = \frac{3\,{V_1}^2 \sqrt{R_1^2 + (X_1 + X_2')^2}}{\omega_S \left\{ \left(R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2} \right)^2 + (X_1 + X_2')^2 \right\}}$$

$$T_m = \frac{3\,{V_1}^2}{2\omega_S \left\{ R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2} \right\}}$$

From the two equations above {(S_m) and (T_m)} it can be seen that (Fig. 14)

- The slip at which maximum torque occurs is proportional to rotor resistance
- The magnitude of the maximum torque is independent of rotor resistance

If all other parameters remain constant, increasing the rotor resistance will:

- Reduce the speed at which maximum torque occurs
- At certain rotor resistance, if it is high enough, we can obtain the maximum torque at starting.

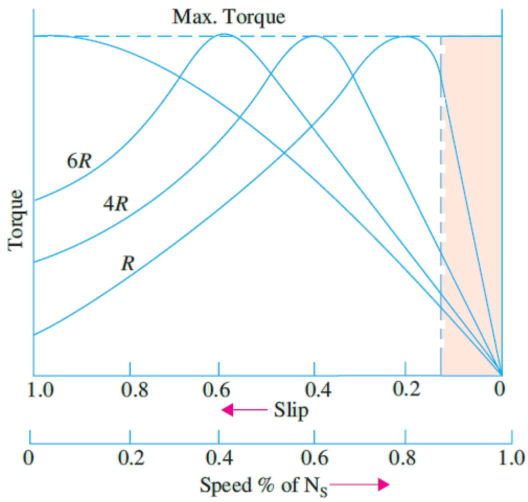


Fig. 14, Torque-speed characteristics at different rotor resistance

Example (10)

A 440-V, 3-φ, 50-Hz, 4-pole, Y-connected induction motor has a full-load speed of 1425 rpm. The rotor has an impedance of (0.4 + J 4) Ω and rotor/stator turns ratio of 0.8. Based on approximate equivalent circuit referred to stator, calculate

***(i)** full-load torque*

***(ii)** rotor current and full-load rotor Cu loss*

***(iii)** power output if windage and friction losses amount to 500 W*

***(iv)** maximum torque and the speed at which it occurs*

***(v)** starting current and*

***(vi)** starting torque.*

Solution

$N_s = 60 \times 50/2 = 1500$ rpm and $\omega_S = 2\pi \times 1500/60 = 157.08$ rad/s

S = (1500 – 1425) / 1500 = 0.05 (full-load slip)

$R_2' = 0.4/(0.8)^2 = 0.625\ \Omega$

$X_2' = 4.0/(0.8)^2 = 6.25\ \Omega$

Rotor impedance $Z_2' = R_2'/S + J X_2' = 12.5 + J6.25 = 13.9754\angle 26.5651\ \Omega$

Rotor current $I_2' = (440/\sqrt{3})/13.9754 = 18.177$ A (full-load current)

$T = 3 (I_2')^2 (R_2'/S)/\omega_s = 3\times(18.177)^2\times 12.5/157.08 = 78.878$ N.m (full-load torque)

Air-gap power (P_g) = $T\times\omega_S = 78.878\times 157.08 = 12.390$ kW

Rotor copper loss (P_{cu2}) = $S\times P_g = 0.05\times 12390 = 619.51$ W

Mechanical power (Gross) = $P_g(1\text{-}S) = 12390\ (1\text{-}0.05) = 11.77$ kW

Mechanical power (Net) = 11770 – 500 = 11.27 kW

If the stator impedance is neglected

$$S_m = \frac{R_2'}{X_2'} = \frac{0.625}{6.25} = 0.1$$

Rotor speed (N) at maximum torque = 1500×(1-0.1) = 1350 rpm

At maximum torque,

Rotor impedance $Z_2' = R_2'/S + J X_2' = 6.25 + J6.25 = 8.839\angle 45$

Rotor Current $I_2' = (440/\sqrt{3})/8.839 = 28.74$ A (Rotor current at T_m)

$T_m = 3\,(I_2')^2\,(R_2'/S)/\omega_s = 3\times(28.74)^2\times6.25/157.08 = 98.6$ N.m

At starting, (S=1)

Rotor impedance $Z_2' = R_2'/S + J\,X_2' = 0.625 + J6.25 = 6.2812\angle 84.29\ \Omega$

Rotor Current $I_2' = (440/\sqrt{3})/6.2812 = 40.444$ A (Rotor current at starting)

$T_{st} = 3\,(I_2')^2\,(R_2'/S)/\omega_s = 3\times(40.444)^2\times0.625/157.08 = 19.525$ N.m

7. Relation Between Torques

As we know that, the torque = $3\,(I_2')^2\,(R_2'/S)/\omega_s$, therefore,

$$T_{f.l} = 3\times\ {I'_{2fl}}^2 \times \frac{R'_2}{S_{fl}}/\omega_s$$

$$T_{st} = 3\times\ {I'_{2st}}^2 \times \frac{R'_2}{1}/\omega_s$$

$$T_m = 3\times\ {I'_{2m}}^2 \times \frac{R'_2}{S_m}/\omega_s$$

From the above equations, we can obtain the following torque relation:

$$\frac{T_m}{T_{f.l}} = \left(\frac{I'_{2m}}{I'_{2fl}}\right)^2 \frac{S_{fl}}{S_m}$$

$$\frac{T_{st}}{T_{f.l}} = \left(\frac{I'_{2st}}{I'_{2fl}}\right)^2 S_{fl}$$

By knowing the slips, we can find another set of relations

$$\frac{T_{fl}}{T_m} = \frac{2\,S_{fl}\,S_m}{S_{fl}^2 + S_m^2}$$

$$\frac{T_{st}}{T_m} = \frac{2\,S_m}{1 + S_m^2}$$

Example (11)

For a 3-phase induction motor having a full-load slip of 3%, the stator impedance and the referred rotor impedance are identical. Also, in each impedance, the leakage reactance is five times the resistance. Determine the P.U. value of the starting torque and maximum torque.

$S_{f.l} = 0.03$ Also $Z_1 = Z_2'$ This means $R_1 + jX_1 = R_2' + JX_2'$

By comparing, $R_1 = R_2'$ and $X_1 = X_2'$ Also $X_1 = 5\ R_1$ and $X_2' = 5R_2'$

It is better to express all parameters as a function of R_2'

then, $R_1 = R_2'$ & $X_1 = 5R_2'$ & $X_2' = 5R_2'$

Considering the approximate equivalent circuit,

$$I_2'^2 = \frac{V_1^2}{\left(R_1 + \frac{R_2'}{S}\right)^2 + (X_1 + X_2')^2}$$

$$T = \frac{3\ V_1^2\ \frac{R_2'}{S}}{\omega_S \left\{\left(R_1 + \frac{R_2'}{S}\right)^2 + (X_1 + X_2')^2\right\}}$$

substituting all parameters with R_2'

$$T = \frac{3\ V_1^2\ \frac{R_2'}{S}}{\omega_S \left\{\left(R_2' + \frac{R_2'}{S}\right)^2 + (5R_2' + 5R_2')^2\right\}} = \frac{3\ V_1^2\ \frac{R_2'}{S}}{R_2'^2 \omega_S \left\{\left(1 + \frac{1}{S}\right)^2 + 100\right\}}$$

$$T = \frac{3\ V_1^2}{S\ R_2'\ \omega_S \left\{\left(1 + \frac{1}{S}\right)^2 + 100\right\}}$$

At Full load

$$T_{f.l} = \frac{3\ V_1^2}{0.03\ R_2'\ \omega_S \left\{\left(1 + \frac{1}{0.03}\right)^2 + 100\right\}} = \frac{3\ V_1^2}{R_2'\ \omega_S} \frac{1}{38.36333}$$

At Starting

$$T_{st} = \frac{3\ V_1^2}{R_2'\ \omega_S \left\{\left(1 + \frac{1}{1}\right)^2 + 100\right\}} = \frac{3\ V_1^2}{R_2'\ \omega_S} \frac{1}{104}$$

Dividing the above two equation, we can get

$$\frac{T_{st}}{T_{fl}} = \frac{38.36333}{104} = 0.3689\ P.U.$$

The maximum torque occurs at S_m

$$S_m = \frac{R_2'}{\sqrt{(R_1)^2 + (X_1 + X_2')^2}} = \frac{R_2'}{\sqrt{(R_2')^2 + (5R_2' + 5R_2')^2}} = \frac{1}{\sqrt{101}} = 0.0995$$

$$T_{max} = \frac{3\,{V_1}^2}{0.0995\,R_2'\,\omega_S\left\{\left(1+\frac{1}{0.0995}\right)^2 + 100\right\}} = \frac{3\,{V_1}^2}{R_2'\,\omega_S}\frac{1}{22.1}$$

Dividing the two equations of T_{max} and T_{fl}, we can get

$$\frac{T_{max}}{T_{fl}} = \frac{38.36333}{22.1} = 1.736\,P.U.$$

Example (12)

A 3-phase, Y-Y connected wound-rotor induction motor has 0.06Ω rotor resistance and 0.3 Ω standstill reactance per phase. Find the additional resistance required in the rotor circuit to make the starting torque equal to the maximum torque.

After inserting an extra rotor resistance Δr we get that

$$\frac{T_{st}}{T_m} = \frac{2\,S_m}{1 + S_m^2} = 1$$

then,

$$S_m^2 - 2S_m + 1 = 0$$

Then $S_m = 1$

but as the stator impedance is neglected

$$S_m = \frac{R_2'}{X_2'}$$

Then $R_2' = X_2' = 0.3$

this means, $R_2' + \Delta r = 0.3$

$\Delta r = 0.3 - 0.06 = 0.24\ \Omega$

Example (13)

A 6-pole, 3-phase, 50-Hz induction motor has rotor resistance and reactance of 0.02 Ω and 0.1 Ω respectively per phase. At what speed would it develop maximum torque?

Find out the value of extra resistance necessary to give half of maximum torque at starting.

N_S = 60×50/3 = 1000 rad/s

Since the stator impedance is neglected, then

$$S_m = \frac{R_2'}{X_2'} = \frac{0.02}{0.1} = 0.2$$

The rotor speed at max. torque = N_s (1-S_m) = 1000(1-0.2) = 800 rad/s ##

After inserting an extra rotor resistance Δr we get that

$$\frac{T_{st}}{T_m} = \frac{2\,S_m}{1 + S_m^2} = \frac{1}{2}$$

$$S_m^2 - 4S_m + 1 = 0$$

Then S_m = 0.26795 (accepted) or S_m = 3.732 (rejected)

$$S_m = 0.26795 = \frac{R_2'}{0.1}$$

Then R_2' = 0.026795 Ω

Then Δr = 0.026795 - 0.02 = 0.006795 Ω ##

Example (14):

A 3-phase, 460-V, 60-Hz, 4-pole, 500-hp wound-rotor induction motor has the following properties:

Rotor current at 20% slip = 3.95 the full load rotor current,
Rotor current at maximum torque = 2.82 the full-load rotor current,
Torque at 20% slip = 1.2 the full-load torque,
Slip at maximum torque = 6%

Calculate:

a) The full-load slip
b) The maximum torque in P.U.
c) The starting current in P.U.
d) The starting torque in P.U.

Rotor current at 20% slip = 3.95 the full load rotor current, → $I_{20\%}$ = 3.95 P.U.

Rotor current at maximum torque = 2.82 the full-load rotor current, → I_m=2.82 P.U.

Torque at 20% slip = 1.2 the full-load torque, → $T_{20\%}$ = 1.2 P.U.

Slip at maximum torque = 6% → S_m = 0.06

$$\frac{T_{20\%}}{T_{fl}} = 1.2 = \left(\frac{I_{20\%}}{I_{fl}}\right)^2 \frac{S_{fl}}{S_{20\%}} = (3.95)^2 \frac{S_{fl}}{0.2}$$

$$S_{fl} = 0.0154 \quad \#$$

$$\frac{T_m}{T_{fl}} = \left(\frac{I_m}{I_{fl}}\right)^2 \frac{S_{fl}}{S_m} = (2.82)^2 \frac{0.0154}{0.06} = 2.041116$$

T_m = 2.041116 P.U. #

$$\frac{T_{fl}}{T_m} = \frac{2\, S_m S_{fl}}{S_m^2 + S_{fl}^2} = \frac{2 \times 0.06 \times 0.0154}{0.06^2 + 0.0154^2} = 0.4816$$

$$\frac{T_{st}}{T_m} = \frac{2\, S_m}{S_m^2 + 1} = \frac{2 \times 0.06}{0.06^2 + 1} = 0.11957$$

T_{st} = 0.11957 × T_m = 0.11957 × T_{fl}/ 0.4816 = 0.2483

T_{st} = 0.2483 P.U. #

$$\frac{T_{st}}{T_{fl}} = 0.2483 = \left(\frac{I_{st}}{I_{fl}}\right)^2 \frac{S_{fl}}{1} = \left(\frac{I_{st}}{I_{fl}}\right)^2 0.0154 \quad \rightarrow \qquad I_{st} = 4.015 \text{ P.U.} \quad \#$$

8. NEMA Design (Induction Motor Classes)

NEMA (National Electrical Manufacturers Association) have classified electrical induction motors into four different NEMA Classes where torques and starting-load inertia are important criterions as shown in Fig 15. This figure illustrates typical speed torque curves for Class A, B, C, and D motors.

- Class A motors have a higher breakdown torque than Class B motors and are usually designed for a specific use. Slip is 5%, or less.

- Class B motors account for most of the induction motors sold. Often referred to as general purpose motors, slip is 5% or less.
- Class C motors have high starting torque with normal starting current and low slip. This design is normally used where breakaway loads are high at starting, but normally run at rated full load, and are not subject to high overload demands after running speed has been reached. Slip is 5% or less.
- Class D motors exhibit high slip (5 to 13%), very high starting torque, low starting current, and low full load speed. Because of high slip, speed can drop when fluctuating loads are encountered. This design is subdivided into several groups that vary according to slip or the shape of the speed-torque curve. These motors are usually available only on a special order basis.

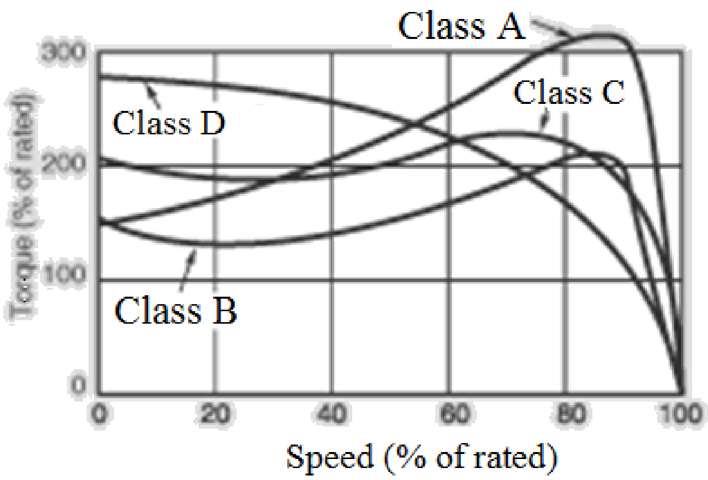


Fig. 15, Different NEMA design of Induction Motors

9. Determination of Induction Machine Parameters

The equivalent circuit parameters for an induction motor can be determined using specific tests on the motor, just as was done for the transformer.

There are two types of tests: Blocked-rotor test that used to determine R_1, R_2', X_1 and X_2' also no load test that used to determine R_c and X_m

DC test can be used to determine the value of the stator resistance R_1

9.1 Blocked Rotor (Short-Circuit) Test

The rotor is blocked to prevent rotation and balanced voltages are applied to the stator terminals at a frequency of 25 percent of the rated frequency (f_{test}) at a reduced voltage such that the rated current is achieved. Current, voltage and power are measured at the motor input.

Since the rotor is standstill, the slip is unity. Thus, the value of the load $R_2'(1-S)/S$ is zero and the rotor circuit is considered as short circuit and the rotor current is much larger than the magnetizing current, so the magnetizing branch can be neglected as shown in Fig. 16. On the other hand, since the slip is unity, then the rotor frequency equals the stator frequency (f_{test}).

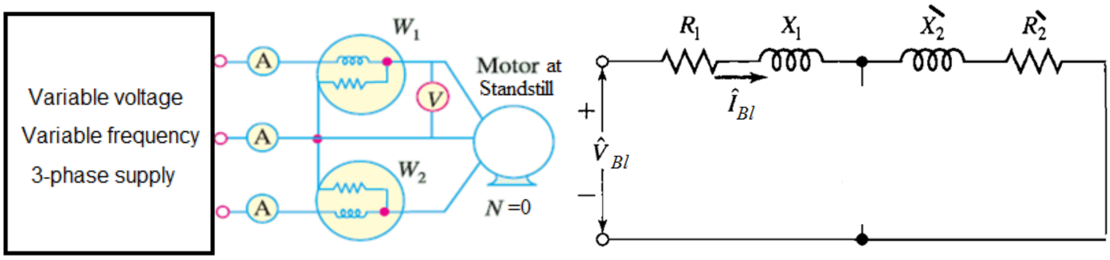


Fig. 16, Induction motor equivalent circuit at blocked rotor test

The power (P_{Bl}) & the voltage (V_{Bl}) and current (I_{Bl}) at blocked rotor test are measured.

The motor impedance Z_{Bl} can be calculated from

$$Z_{Bl} = \frac{V_{Bl}}{I_{Bl}}$$

The motor power factor can be calculated as

$$\cos(\varphi_{Bl}) = \frac{P_{Bl}}{3\, V_{Bl} I_{Bl}}$$

The stator and rotor resistances can be calculated as:

$$R_1 + R_2' = Z_{Bl} \cos(\varphi_{Bl})$$

Assuming R_1 and R_2' are equal

$$R_1 = R_2' = \frac{Z_{Bl} \cos(\varphi_{Bl})}{2}$$

The stator and rotor reactances can be calculated as

$$X_{Bl} = X_1 + X_2' = Z_{Bl} \sin(\varphi_{Bl})$$

According to the motor class X_1 and X_2' can be calculated as

Squirrel-cage Class A	$X_1 = 0.5X_{Bl}$	$X_2 = 0.5\ X_{Bl}$
Squirrel-cage Class B	$X_1 = 0.4\ X_{Bl}$	$X_2 = 0.6\ X_{Bl}$
Squirrel-cage Class C	$X_1 = 0.3\ X_{Bl}$	$X_2 = 0.7\ X_{Bl}$
Squirrel-cage Class D	$X_1 = 0.5\ X_{Bl}$	$X_2 = 0.5\ X_{Bl}$
Wound rotor	$X_1 = 0.5\ X_{Bl}$	$X_2 = 0.5\ X_{Bl}$

9.2 No-Load (Open-Circuit) Test

Balanced voltages are applied to the stator terminals at the rated frequency with the rotor uncoupled from any mechanical load. Current, voltage and power are measured at the motor input. The losses in the no-load test are those due to core losses, copper losses, windage and friction losses.

The slip of the induction motor at no-load is very small. Thus, the value of the load resistance $R_2'(1-S)/S$ is very high and the rotor circuit is considered as open circuit as shown in Fig. 17.

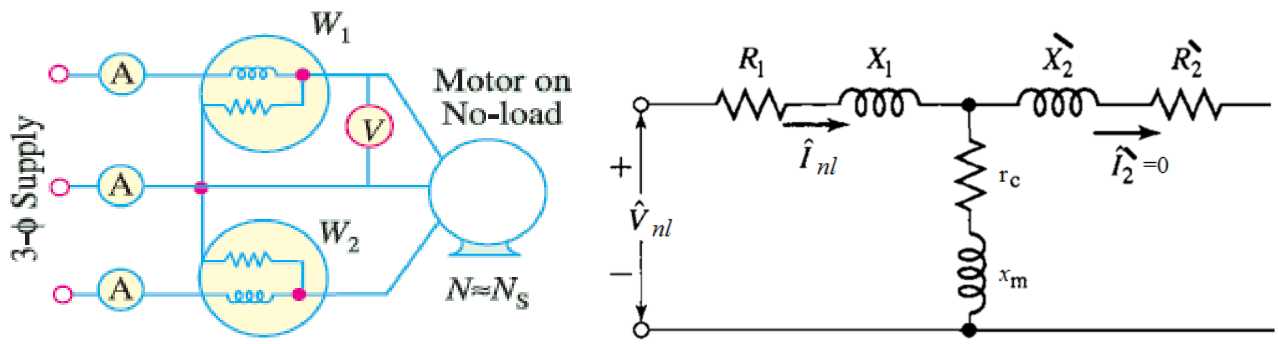


Fig. 17, motor equivalent circuit at no load

The input power measured in the no-load test is equal to the stator copper losses plus the core loss plus the rotational losses.

The power (P_{Nl}) & the voltage (V_{Nl}) and current (I_{Nl}) at No-Load test are measured.

The motor impedance Z_{Nl} can be calculated from

$$Z_{Nl} = \frac{V_{Nl}}{I_{Nl}}$$

The motor power factor can be calculated as

$$\cos(\varphi_{Nl}) = \frac{P_{Nl}}{3\ V_{Nl} I_{Nl}}$$

The stator and core resistances can be calculated as:

$$R_1 + r_c = Z_{Nl} \cos(\varphi_{Nl})$$

But the stator resistance R_1 is calculated from blocked rotor test

$$r_c = Z_{Nl} \cos(\varphi_{Nl}) - R_1$$

The stator and magnetizing reactances can be calculated as:

$$X_1 + x_m = Z_{Nl} \sin(\varphi_{Nl})$$

The stator reactance is calculated previously from the blocked-rotor test

$$x_m = Z_{Nl} \sin(\varphi_{Nl}) - X_1$$

Example (15)

The test data applied to a three-phase, Delta-connected, 220-V, 20-A, 60-Hz, four-pole, wound-rotor induction motor are:

The No-load test at 60 Hz No-load current = 5.90 A and Power P_{NL} =310 W

The Blocked-rotor test at 15 Hz

Applied voltage is 20% of the rated voltage

Blocked-Rotor current = rated armature current, and Power P_{BL} = 575 W

DC test V_{DC} = 13.62 V, and I_{DC} = 30 A. Calculate

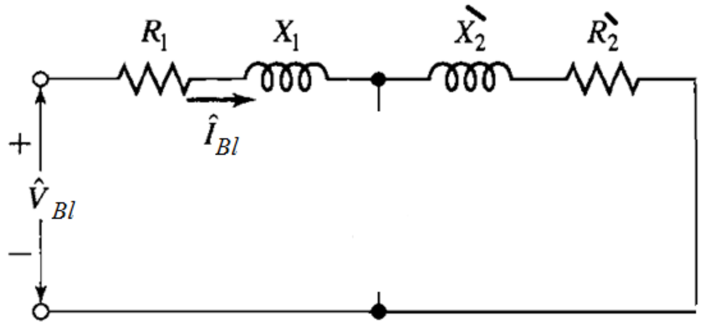
a) Equivalent-circuit parameters applying to the normal running conditions. Then calculate the no-load iron loss and stator cu loss.
b) when this motor is operating at rated voltage and frequency and at a slip of 3%, calculate the air-gap power, the maximum torque and the corresponding slip.

From DC Test:

$R_{DC} = V_{DC}/I_{DC} = 0.454\ \Omega$

Since the stator is delta connected, $R_{1DC} = (3/2)\ R_{DC} = 0.681\Omega$

From Blocked rotor Test:

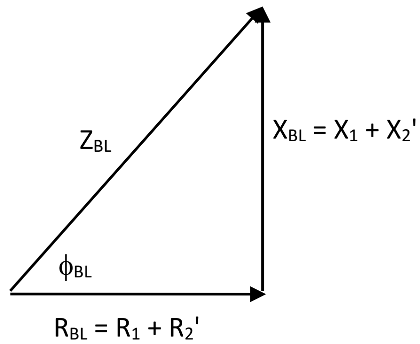


V_{BL} = 0.2×220 = 44 V

I_{BL} = 20/√3 = 11.547 A

P_{BL} = 575 = 3 × 44 × 11.547 × cos(ϕ_{BL})

$$\cos(\phi_{BL}) = \frac{575}{3 \times 44 \times 11.547} = 0.37725$$



ϕ_{BL} = 67.8368°

sin(ϕ_{BL}) = 0.92611

$Z_{BL} = V_{BL}/I_{BL}$ = 44/11.547 = 3.8105Ω

$R_{BL} = Z_{BL}\cos(\phi_{BL})$ = 1.438Ω

Since the frequency is 15Hz, this corresponds to 5% increment in stator resistance due to skin effect. Therefore,

R_1 = 0.681 × 1.05 = 0.715Ω

$R_2' = R_{BL} – R_1$ = 0.723Ω

$X_{BL} = Z_{BL}\sin(\phi_{BL})$ = 3.529Ω

Since the motor is wound type, then $X_1 = X_2' = X_{BL}/2$ = 1.765Ω

Since the no load test is done at 60 Hz, the motor parameters must be adjusted to meet the operation at 60 Hz as follows:

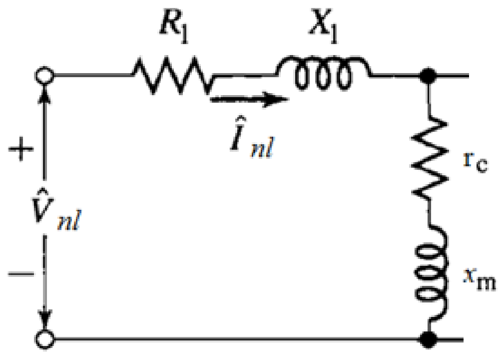
The resistances were calculated in blocked rotor test at 15 Hz. Therefore, the skin effect at 60 Hz is considered as 15%:

$$R_1 = \frac{0.715}{1.05} 1.15 = 0.7831\Omega$$

$$R'_2 = \frac{0.723}{1.05} 1.15 = 0.792\Omega$$

$$X_1 = X'_2 = \frac{1.765}{15} 60 = 7.06\Omega$$

<u>From No-Load Test:</u>



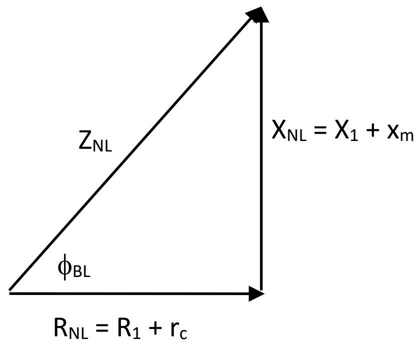
$V_{NL} = 220$ V

$I_{NL} = 5.9/\sqrt{3} = 3.4064$ A

$P_{NL} = 310 = 3 \times 220 \times 3.4064 \times \cos(\phi_{NL})$

$$\cos(\phi_{NL}) = \frac{310}{3 \times 220 \times 3.4064} = 0.137887$$

$\phi_{NL} = 82.0744°$ and $\sin(\phi_{NL}) = 0.99044$



$Z_{NL} = V_{NL}/I_{NL} = 220/3.4064 = 64.584\Omega$

$R_{NL} = Z_{NL} \cos(\phi_{NL}) = 8.9053\Omega$

$r_c = 8.9053 – 0.7831 = 8.122\Omega$

$X_{NL} = Z_{NL} \sin(\phi_{NL}) = 63.9666\ \Omega$

$X_m = 63.9666 – 7.06 = 56.91\Omega$

The motor parameters are summarized as:

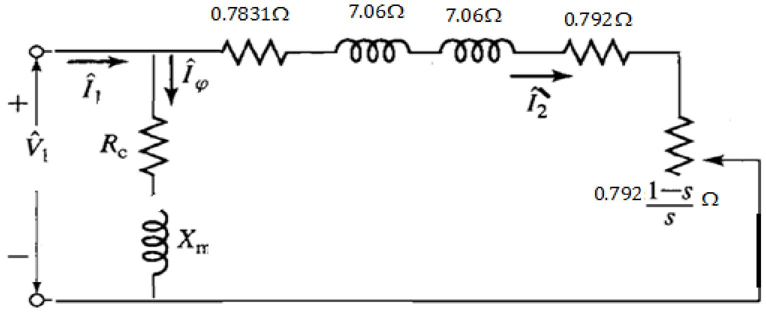
R1	**R2'**	**X1**	**X2'**	**r_c**	**x_m**
0.78321Ω	0.792Ω	7.06Ω	7.06Ω	8.122Ω	56.91Ω

<u>at no-load,</u>

$P_{cu1} = 3(I_{NL})^2 R_1 = 3 \times (3.4064)^2 \times 0.78321 = 27.26$ W

$P_{iron} = 3(I_{NL})^2 r_c = 3 \times (3.4064)^2 \times 8.122 = 282.732$ W

b) The overall approximate equivalent circuit at S = 0.03 is shown



Z = (0.7831+25.608) + J(7.06+7.06) = 26.3911 + J 14.12 = 29.931∠28.148Ω

I_2' = V_1/Z = 220/29.931∠28.148 = 7.35∠ − 28.148 A

Air-gap power (P_g) = 3 $I_2'^2$ R_2'/S = 3×$(7.35)^2$×0.792/0.03 = 4278.582 W

N_s = 60 × 60/2 = 1800 rpm

ω_s = 2π N_s/60 = 2π 1800/60 =188.496

$$T_m = \frac{3\,{V_1}^2}{2\omega_S\left\{R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}\right\}}$$

$$T_m = \frac{3\,(220)^2}{2 \times 188.496\left\{0.7831 + \sqrt{(0.7831)^2 + (14.12)^2}\right\}} = 25.806\ N.m$$

$$S_m = \frac{R_2'}{\sqrt{(R_1)^2 + (X_1 + X_2')^2}}$$

$$S_m = \frac{0.792}{\sqrt{(0.7831)^2 + (14.12)^2}} = 0.056$$

Example (16)

The test data applied to a three-phase, Star-connected, 440-V, 20-A, 60-Hz, four-pole, class "C" induction motor are:

	Frequency (Hz)	Voltage (V)	Current (A)	Power (W)
No-Load Test	60	Rated	5.9	410
Blocked-Rotor Test	15	25% of rated	Rated	618
DC Test	--	16.8	35	--

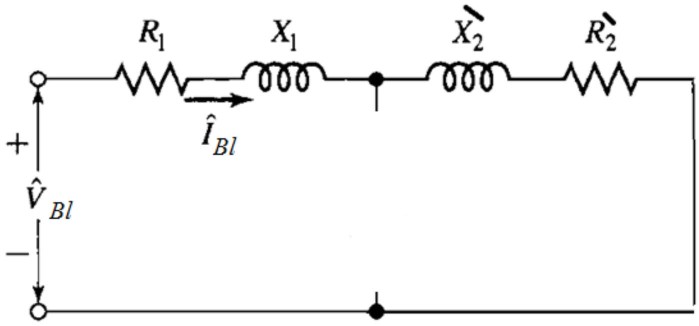
Determine the equivalent circuit parameters applying to the normal running conditions. Then calculate the no-load iron loss and stator cu loss.

From DC Test:

R_{DC} = V_{DC}/I_{DC} = 16.8/35 = 0.48 Ω

Since the stator is star connected, R_{1DC} = (1/2) R_{DC} = 0.24 Ω

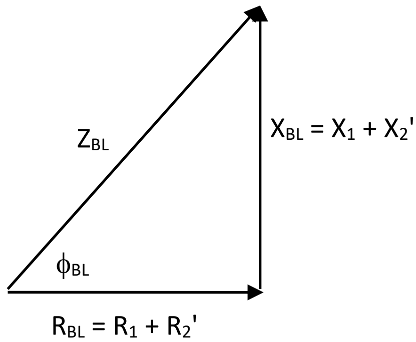
From Blocked rotor Test:



$V_{BL} = 0.25 \times 440/\sqrt{3} = 63.51$ V

$I_{BL} = 20$ A

$P_{BL} = 618 = 3 \times 63.51 \times 20 \times \cos(\phi_{BL})$



$$\cos(\phi_{BL}) = \frac{618}{3 \times 63.51 \times 20} = 0.16218$$

$\phi_{BL} = 80.6666°$

$\sin(\phi_{BL}) = 0.98676$

$Z_{BL} = V_{BL}/I_{BL} = 63.51/20 = 3.1755\ \Omega$

$R_{BL} = Z_{BL} \cos(\phi_{BL}) = 0.515\ \Omega$

Since the frequency is 15Hz, this corresponds to 5% increment in stator resistance due to skin effect. Therefore,

$R_1 = 0.24 \times 1.05 = 0.252\ \Omega$

$R_2' = R_{BL} - R_1 = 0.515 - 0.252 = 0.263\ \Omega$

$X_{BL} = Z_{BL} \sin(\phi_{BL}) = 3.1335\ \Omega$

Since the motor is class "C", then

$X_1 = 0.3\ X_{BL} = 0.3 \times 3.1335 = 0.94\ \Omega$

$X_2' = 0.7\ X_{BL} = 0.7\times 3.1335 = 2.19345\ \Omega$

Since the no load test is done at 60 Hz, the motor parameters must be adjusted to meet the operation at 60 Hz as follows:

The resistances were calculated in blocked rotor test at 15 Hz. Therefore, the skin effect at 60 Hz is considered as 15%:

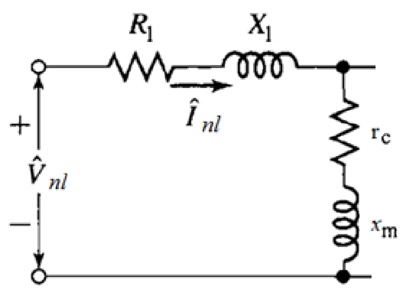
$$R_1 = \frac{0.252}{1.05}1.15 = 0.276\Omega$$

$$R'_2 = \frac{0.263}{1.05}1.15 = 0.288\Omega$$

$$X_1 = \frac{0.94}{15}60 = 3.76\ \Omega$$

$$X'_2 = \frac{2.19345}{15}60 = 8.7738\ \Omega$$

From No-Load Test:



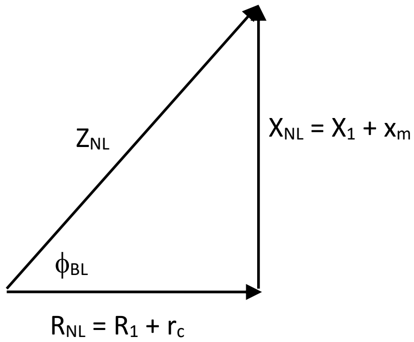
$V_{NL} = 440/\sqrt{3} = 254.034V$

$I_{NL} = 5.9\ A$

$P_{NL} = 410 = 3 \times 254.034 \times 5.9 \times \cos(\phi_{NL})$

$$\cos(\phi_{NL}) = \frac{410}{3\times 254.034\times 5.9} = 0.0912$$

$\phi_{NL} = 84.7683°$ and $\sin(\phi_{NL}) = 0.9958$



$Z_{NL} = V_{NL}/I_{NL} = 254.034/5.9 = 43.0566\ \Omega$

$R_{NL} = Z_{NL}\cos(\phi_{NL}) = 3.9268\ \Omega$

$r_c = 3.9268 - 0.276 = 3.6508\ \Omega$

$X_{NL} = Z_{NL} \sin(\phi_{NL}) = 42.8758\ \Omega$

$X_m = 42.8758 - 3.76 = 39.1158\ \Omega$

The motor parameters are summarized as:

R1	R2'	X1	X2'	r_c	x_m
0.276Ω	0.288Ω	3.76 Ω	8.7738Ω	3.6508Ω	39.1158Ω

at no-load,

$P_{cu1} = 3(I_{NL})^2 R_1 = 3 \times (5.9)^2 \times 0.276 = 28.823$ W

$P_{iron} = 3(I_{NL})^2 r_c = 3 \times (5.9)^2 \times 3.6508 = 381.253$ W

Example (17):

The following data apply to a 125-kW, 2300-V, 3-phase, Y-connected, 4-pole, 60-Hz squirrel-cage class B, induction motor:

DC Test: *DC resistance between stator terminals is 2.24 Ω*

No-load test (60 Hz): *Line current = 7.7 A, Three-phase power = 2870 W*

Blocked-rotor test (15 Hz): *Line current = 50.3 A, and Three-phase power = 18.2 kW*

a) Calculate the equivalent-circuit parameters in ohms.

b) Based on approximate equivalent circuit, compute the rotor current, input power factor, air-gap power and the maximum slip when the motor is operating at rated voltage & frequency at slip of 2.95 %.

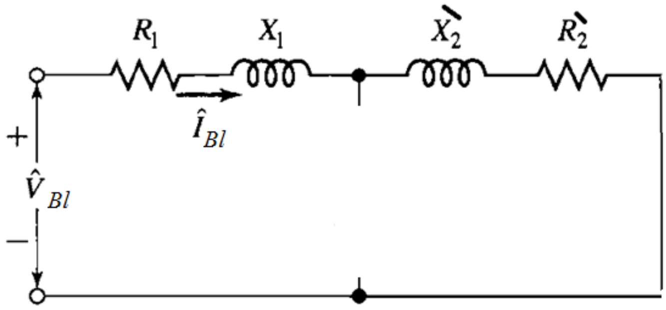
(Consider the skin effect at 60 Hz is 20% and at 15 Hz is 7%)

From DC Test:

$R_{DC} = V_{DC}/I_{DC} = 2.24\ \Omega$

Since the stator is star connected, $R_{1DC} = (1/2)\ R_{DC} = 1.12\ \Omega$

From Blocked rotor Test:

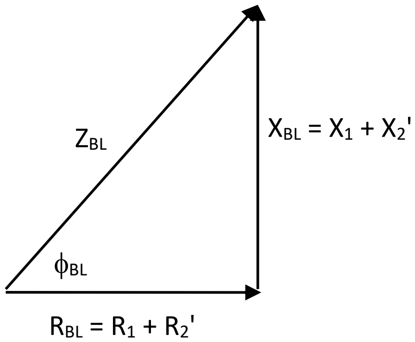


$V_{BL} = 0.25\times2300/\sqrt{3} = 331.9764$ V

$I_{BL} = 50.3$ A

$P_{BL} = 18200 = 3 \times 331.9764 \times 50.3 \times \cos(\phi_{BL})$

$$\cos(\phi_{BL}) = \frac{18200}{3 \times 331.9764 \times 50.3} = 0.36331$$



$\phi_{BL} = 68.69651°$

$\sin(\phi_{BL}) = 0.93167$

$Z_{BL} = V_{BL}/I_{BL} = 331.9764/50.3 = 6.6\ \Omega$

$R_{BL} = Z_{BL} \cos(\phi_{BL}) = 2.39785\ \Omega$

Since the frequency is 15Hz, this corresponds to 7% increment in stator resistance due to skin effect. Therefore,

$R_1 = 1.12 \times 1.07 = 1.1984\ \Omega$

$R_2' = R_{BL} - R_1 = 2.39785 - 1.1984 = 1.2\ \Omega$

$X_{BL} = Z_{BL} \sin(\phi_{BL}) = 6.149022\ \Omega$

Since the motor is class "B", then

$X_1 = 0.4\ X_{BL} = 0.4\times 6.149022 = 2.45961\ \Omega$

$X_2' = 0.6\ X_{BL} = 0.6\times 6.149022 = 3.68941\ \Omega$

Since the no load test is done at 60 Hz, the motor parameters must be adjusted to meet the operation at 60 Hz as follows:

The resistances were calculated in blocked rotor test at 15 Hz. Therefore, the skin effect at 60 Hz is considered as 20%:

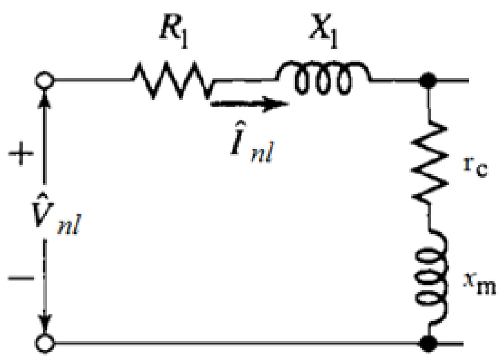
$$R_1 = \frac{1.1984}{1.07} 1.2 = 1.344\ \Omega$$

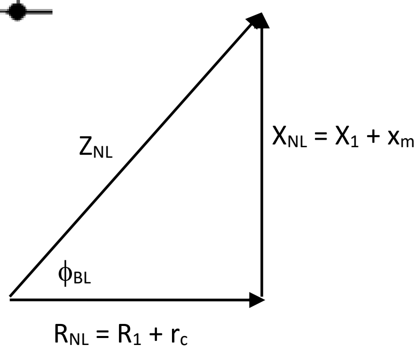
$$R'_2 = \frac{1.2}{1.07} 1.2 = 1.3458\ \Omega$$

$$X_1 = \frac{2.45961}{15} 60 = 9.83844\ \Omega$$

$$X'_2 = \frac{3.68941}{15} 60 = 14.75764\ \Omega$$

From No-Load Test:





$V_{NL} = 2300 / \sqrt{3} = 1327.91V$

$I_{NL} = 7.7$ A

$P_{NL} = 2870 = 3 \times 1327.91 \times 7.7 \times \cos(\phi_{NL})$

$$\cos(\phi_{NL}) = \frac{2870}{3 \times 1327.91 \times 7.7} = 0.0935624$$

$\phi_{NL} = 84.63142°$ and $\sin(\phi_{NL}) = 0.99561342$

$Z_{NL} = V_{NL}/I_{NL} = 1327.91/7.7 = 172.455844\ \Omega$

$R_{NL} = Z_{NL} \cos(\phi_{NL}) = 16.135383\ \Omega$

$r_c = R_{NL} - R_1 = 16.135383 - 1.344 = 14.7914\ \Omega$

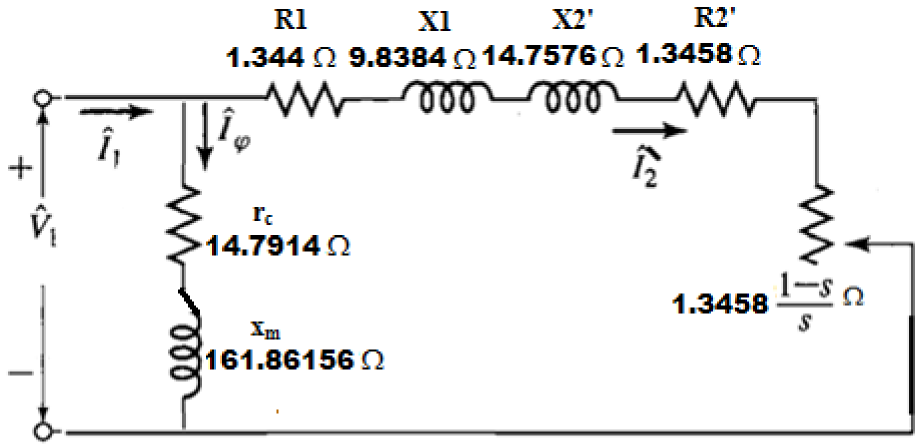
$X_{NL} = Z_{NL} \sin(\phi_{NL}) = 171.7\ \Omega$

$x_m = X_{NL} - X_1 = 171.7 - 9.83844 = 161.86156\ \Omega$

The motor parameters are summarized as:

R1	R2'	X1	X2'	r_c	x_m
1.344 Ω	1.3458 Ω	9.83844 Ω	14.75764 Ω	14.7914 Ω	161.86156 Ω

b) The overall approximate equivalent circuit at S = 0.0295 is shown



Z = (1.344+J9.83844) + (45.62034+J14.75764) = 46.96434 + J 24.59608 Ω

= 53.01525∠27.64187 Ω

$$I_2{}' = \frac{V}{Z} = \frac{2300/\sqrt{3}}{53.01525\angle 27.64187} = 25.04762\angle -27.64187\ A$$

Air-gap power (P_g) = 3 $I_2'^2$ R_2'/S = 3×(25.04762)^2×1.3458/0.0295 = 85864.312 W

N_s = 60 × 60/2 = 1800 rpm

ω_s = 2π N_s/60 = 2π 1800/60 =188.496

$$T_{f.l} = \frac{P_g}{\omega_s} = \frac{85864.312}{188.496} = 455.523\ N.m$$

$$T_m = \frac{3\ V_1{}^2}{2\omega_S\left\{R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}\right\}}$$

$$T_m = \frac{3\ (1327.91)^2}{2\times 188.496\left\{1.344 + \sqrt{(1.344)^2 + (24.59608)^2}\right\}} = 540.18334\ N.m$$

$$S_m = \frac{R_2'}{\sqrt{(R_1)^2 + (X_1 + X_2')^2}}$$

$$S_m = \frac{1.3458}{\sqrt{(1.344)^2 + (24.59608)^2}} = 0.0546$$

Example (18):

A 460-V, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following parameters in ohms per phase referred to the stator circuit:

R_1 = 0.641ΩR_2' = 0.332 Ω X$_1$ = 1.106 Ω X$_2'$ = 0.464 Ω X$_m$= 26.3 Ω

Based on approximate equivalent circuit, and neglecting the iron loss:

a) What is the maximum torque of this motor? At what speed and slip does it occur?

b) What is the starting torque of this motor?

c) If the rotational loss is 350W, what is the full-load efficiency? Consider $S_{f.l}$=0.05

d) If the rotor resistance is doubled, what is the maximum torque and max slip?

e) What is the new starting torque of the motor?

a) the maximum slip:

$$S_m = \frac{R_2'}{\sqrt{(R_1)^2 + (X_1 + X_2')^2}} = \frac{0.332}{\sqrt{(0.641)^2 + (1.57)^2}} = 0.1958$$

$N_s = 60\times f / P = 60\times 60 / 2 = 1800$ rpm

** The rotor speed (N) = N_s (1-S) = 1800 (1-0.1958) = 1447.6 rpm #

ω_s = 188.496 rad/s

$$T_m = \frac{3\, {V_1}^2}{2\omega_S \left\{R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}\right\}}$$

$$= \frac{3 \times (265.5811)^2}{2 \times 188.496 \left\{0.641 + \sqrt{(0.641)^2 + (1.57)^2}\right\}} = 240.1925\ N.m$$

b) At starting, S=1

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + \frac{R_2'}{S}\right)^2 + (X_1 + X_2')^2}} = \frac{265.5811}{\sqrt{(0.973)^2 + (1.57)^2}} = 143.7859\ A$$

$$T_{st} = \frac{3\, {I_2'}^2 \frac{R_2'}{S}}{\omega_S} = \frac{3 \times (143.7859)^2 \times 0.332}{188.496} = 109.242\ N.m$$

c) At full load, S=0.05

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + \frac{R_2'}{S}\right)^2 + (X_1 + X_2')^2}} = \frac{265.5811}{\sqrt{(7.281)^2 + (1.57)^2}} = 35.6564\ A$$

$$P_g = 3\ {I_2'}^2 \frac{R_2'}{S} = 3 \times (35.6564)^2 \times \frac{0.332}{0.05} = 25325.867\ W$$

$$P_{mech}(Gross) = 3\ {I_2'}^2 \frac{R_2'(1-S)}{S} = 3 \times (35.6564)^2 \times \frac{0.332(1-0.05)}{0.05}$$
$$= 24059.5736\ W$$

$$P_{mech}(Net) = 24059.5736 - 350 = 23709.5736\ W$$

$$P_{cu1} = 3\ {I_2'}^2 R_1 = 3 \times (35.6564)^2 \times 0.641 = 2444.8616\ W$$

$$\eta = \frac{23709.5736}{25325.867 + 2444.8616} \times 100 = 85.3761\%$$

d) The rotor resistance is doubled $R_2' = 0.664\ \Omega$

$$S_m = \frac{R_2'}{\sqrt{(R_1)^2 + (X_1 + X_2')^2}} = \frac{0.664}{\sqrt{(0.641)^2 + (1.57)^2}} = 0.3916$$

Max. torque is not changed T_m = 240.1925 N.m

e) At starting, S=1

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + \frac{R_2'}{S}\right)^2 + (X_1 + X_2')^2}} = \frac{265.5811}{\sqrt{(1.305)^2 + (1.57)^2}} = 130.088\ A$$

$$T_{st} = \frac{3\ {I_2'}^2 \frac{R_2'}{S}}{\omega_S} = \frac{3 \times (130.088)^2 \times 0.664}{188.496} = 178.8387\ N.m$$

Sheet 4(Three-Phase Induction Machines)

Problem (1)

A three-phase, Y-connected, 220-V (line-to-line), 7.5-kW, 60-Hz, 6-pole induction motor has the following parameter values in Ω/phase referred to the stator:

$R_1 = 0.294\Omega$ $R_2' = 0.144\ \Omega$ $R_c = 415\ \Omega$

$X_1 = 0.503\ \Omega$ $X_2' = 0.209\ \Omega$ $X_m = 13.25\ \Omega$

The total friction and windage losses may be assumed to be constant at 403 W, independent of load. For a slip of 2 %, and based on EXACT equivalent circuit, compute the rotor speed, output torque and power, stator current, power factor, iron loss, electro-magnetic torque & efficiency when the motor is operated at rated voltage and frequency.

Problem (2)

A three-phase, four-pole, 30hp, 220V, 60Hz, Y-connected induction motor draws a current of 77A from the line source at a power factor of 0.88. At this operating condition, the motor losses are known to be the following:

Stator copper losses: P_{Cu1}=1,033W

Rotor copper losses: P_{Cu2}=1,299W

Stator core losses: P_{iron}=485W

Rotational losses: P_{f+w}=540W (friction, windage). Determine:

a) power transferred across the air gap,

b) slip expressed per unit and in rpm,

c) mechanical power developed in watts,

d) horsepower output,

e) motor speed in rpm and radians per second,

f) torque at the output shaft,

g) efficiency of operation at the stated condition.

Problem (3)

A three-phase, four-pole, 50hp, 480V, 60Hz, Y-connected induction motor has the following parameters per phase:

$R_c = 190\,\Omega$, $X_m = 15\,\Omega$, $R_1 = 0.10\,\Omega$, $X_1 = 0.35\,\Omega$, $R_2' = 0.12\,\Omega$, $X_2' = 0.40\,\Omega$.

It is known that the rotational losses amount is 950W. Calculate:

a) the starting line current when starting from rest under full line voltage,

b) the input line current under no load conditions,

c) the slip under no load conditions expressed per unit and in rpm.

When the motor operates at a slip of 3.9%, find:

d) the input line current and power factor,

e) the horsepower output,

f) the efficiency.

Problem (4)

A three-phase, Y-connected, 460-V (line-line), 25-kW, 60-Hz, four-pole induction motor has the following equivalent-circuit parameters in ohms per phase referred to the stator:

R_1=0.103 R_2'=0.225 X_1=1.10 X_2'=1.13 X_m=59.4

The total friction and windage losses may be assumed constant at 265 W, and the core loss may be assumed to be equal to 220 W. With the motor connected directly to a 460-V source, compute the speed, output shaft torque and power, input power and power factor and efficiency for slips of 1, 2 and 3 percent.

Hint: You can represent the core loss by a resistance R_c connected in parallel with the magnetizing reactance X_m.

Problem (5)

A 15-kW, 230-V, three-phase, Y-connected, 60-Hz, four-pole squirrel-cage induction motor develops full-load torque at a slip of 3.5 percent when operated at rated voltage and frequency. If the rotational and core losses are neglected. The following motor parameters, in ohms per phase, have been obtained:

R_1 = 0.21 X_1= X_2' = 0.26 X_m = 10.1

Determine the maximum torque at rated voltage and frequency, the slip at maximum torque, and the starting torque at rated voltage and frequency.

Problem (6)

A three-phase induction motor, operating at rated voltage and frequency, has a starting torque of 135 percent and a maximum torque of 220 percent, both with respect to its rated-load torque. Neglecting the effects of stator resistance and rotational losses and assuming constant rotor resistance, determine:

a. the slip at maximum torque.

b. the slip at rated load.

c. the rotor current at starting (as a percentage of rotor current at rated load).

Problem (7)

When operated at rated voltage and frequency, a three-phase squirrel-cage induction motor delivers full load at a slip of 8.7 percent and develops a maximum torque of 230 percent of full load at a slip of 55 percent. Neglect core and rotational losses and assume that the rotor resistance and inductance remain constant, independent of slip. Determine the torque at starting, with rated voltage and frequency, in per unit based upon its full-load value.

Problem (8)

A 50 Hz, 8 pole induction motor has full load slip of 4%. The rotor resistance and stand still reactance are 0.01 and 0.1 Ω per phase respectively. Find:

i) The speed at which maximum torque occurs

ii) The ratio of maximum torque to full load torque

Problem (9)

The following test data apply to a 7.5-hp, 3-phase Y, 220-V, 19-A, 60-Hz, 4-pole induction motor of design class C (high-starting-torque, low-starting current type):

Test 1: No-load test at 60 Hz

Applied Line voltage V = 220 V, No-load current = 5.70 A and Power P_{NL} =380 W

Test 2: Blocked-rotor test at 15 Hz

Applied voltage V = 26.5 V line-to-line

Blocked-Rotor current = 18.57 A and Power P_{BL} = 675 W

Test 3: DC resistance per stator phase (measured immediately after test 2)

R1 = 0.262 Ω

Compute the no-load rotational loss and the equivalent-circuit parameters applying to the normal running conditions. Neglect any effect of core losses.

Problem (10)

The following data apply to a 125-kW, 2300-V, three-phase, four pole, 60-Hz squirrel-cage induction motor:

Stator-resistance between phase terminals = 2.23 Ω

No-load test at rated frequency and voltage:

Line current = 7.7 A, Three-phase power = 2870 W

Blocked-rotor test at 15 Hz:

Line voltage = 268 V, Line current = 50.3 A, and Three-phase power = 18.2 kW

a. Calculate the rotational losses.

b. Calculate the equivalent-circuit parameters in ohms. Assume that $X_l = X_2'$.

c. Compute the stator current, input power and power factor, output power and efficiency when this motor is operating at rated voltage and frequency at a slip of 2.95 percent.